Received: *March 9, 1999*, Accepted: *June 3, 1999*

Composite Two-Higgs Models and Chiral Symmetry Restoration

Alexandr A. Andrianov*

Departament d'Estructura i Constituents de la Matèria,
Universitat de Barcelona, Diagonal, 647, 08028 Barcelona, Spain
E-mail: andrian@ecm.ub.es

Vladimir A. Andrianov†

Department of Theoretical Physics, Institute of Physics,
St.-Petersburg State University, 198904 St.-Petersburg, Russia
E-mail: andrian@hen.gc.spb.ru

Rudolf Rodenberg‡

Abteilung für Theoretische Elementarteilchen Physik, III. Physikalisches Institut,
Physikzentrum, RWTH-Aachen, D-52056, Aachen, Germany
E-mail: rodenberg@acphyz.physik.rwth-aachen.de

Abstract: The effective quark models with quasilocal interaction are used for description of two composite Higgs doublets, in strong coupling (tricritical) regime below the compositeness scale Λ_C. The low energy effective action of Two-Higgs Doublet Standard Model is obtained in the large N_c and large-log approximation. The two-point correlators of scalar and pseudoscalar Higgs fields are derived for investigation of how the chiral symmetry is broken. The comparison of their asymptotics at high energies allows to realize the chiral symmetry restoration characteristic for the underlying gauge theories with vector coupling to fermions. In this way we make hints on the existence of new physical phenomena in the TeV energy region.

Keywords: Technicolor and Composite Models, Higgs Physics.

In Memory of Alexei A. Anselm: This paper is dedicated to the memory of outstanding Russian physicist Alexei A. Anselm whose teaching and influence during many years was indispensable in our penetration into the Higgs phenomenon [1].

*Supported by Grants DFG 436 RUS 113/227/5, RFBR 98-02-18137, GRACENAS 6-19-97 and by Generalitat de Catalunya, Grant PIV 1999.

†Supported by Grants DFG 436 RUS 113/227/5, RFBR 98-02-18137, GRACENAS 97-40-1.6-7

‡Supported by DFG Grant 436 RUS 113/227/5.

JHEP06(1999)003

Contents

1. Introduction: definition of 2HQQM

One of the minimal extensions of the Standard Model (SM) is the Two Higgs Doublet Model (2HDM) [2, 3] which has two complex doublets of Higgs bosons instead of only one in SM. The general 2HDM allows too strong flavour-changing neutral currents (FCNC) [4] as compared to the phenomenology of electroweak decays [5]. One possibility to suppress FCNC is to couple the fermions only to a fixed combination of the two Higgs doublets and to its charge conjugated one, which is known as a Model I [3]. There is another possibility to couple the first Higgs doublet to down-type quarks while the second one to up-type quarks, which is known as a Model II [4]. In the absence of CP violation, the physical content of the Higgs sector includes a pair of CP-even neutral scalar Higgs bosons, H^0 and h^0, a CP-odd neutral pseudoscalar Higgs boson A and a pair of charged Higgs bosons $H^{\pm}$. The mass spectrum of Models I and II has been extensively studied theoretically [2, 3, 6, 7] and bounded from the phenomenology of EW interactions at available high energies (for recent analyses, see [8, 9] and refs. therein).

If Higgs bosons are composite and their masses are created by a mechanism [10] of spontaneous chiral symmetry breaking [11]–[16] (far below a scale of compositeness Λ), one should expect the chiral symmetry restoration (CSR) at high energies [17, 18] which then leads to the CSR constraints on phenomenological parameters of a Higgs model. These constraints on Higgs boson masses (and other parameters) may serve to pin-point the signatures of compositeness in the future experiments on Higgs particle observation [19].

Recently we have developed effective quark models including higher-dimensional operators made of fermion fields and their derivatives, which can be used [20] for the parameterization of unknown heavy particle dynamics beyond SM in the spirit of Wilsonian effective action approach [21]. The inclusion of higher dimensional operators in the fermionic lagrangean of SM opens the ways to built the two (and more) Higgs doublet extension of SM with composite Higgs bosons. We remark that the strictly local four-fermion interaction can be also used to create two doublets of composite Higgs bosons [14, 15, 22, 23]. However it leads rather to a 2HDM of type II whereas the generation of composite states in the vicinity of tricritical point in coupling constants of quasilocal vertices opens quite different and broader windows for the mass spectrum of Higgs bosons.

The purpose of this paper is to explore particular Effective Quasilocal Quark Models (EQQM) [24] which accept relatively light Higgs boson states and conceivably inherit main properties of a underlying vector gauge theory (possibly of QCD type) [25, 26] giving rise to effective four-fermion operators at low energies. The latter theory may be thought of, for instance, as a Topcolor [23, 27] model or an extended QCD with additional heavy gluons (possibly manifesting extra spatial dimensions [26, 29] which induce the effective four-fermion interaction and enforce t-quarks to condensate). The effective potential of EQQM for boson fields and the Higgs boson mass spectrum is derived in sec. 2 where we restrict ourselves with the consideration of EQQM of type I. The lower bound for the h^0 boson mass is found to be $m_{h^0} \geq 120$ GeV for the t-quark mass $m_t \simeq 175$ GeV whereas the pseudoscalar A bosons can be as light as it is allowed by the experimental bounds.

The most important property of EQQM turns out to be the Chiral Symmetry Breaking (CSB) at low energies and, on the other hand, the Chiral Symmetry Restoration (CSR) at high energies. In the underlying theory it is controlled by the Operator Product Expansion (OPE) of quark current correlators which include a different number of parity-odd and parity-even currents. More specifically, here we deal with the two-point correlators of scalar and pseudoscalar quark densities [18, 28] which are saturated at low energies by Higgs-particle resonances of a definite parity. The difference between these correlators is decreasing rapidly in accordance to OPE of a vector-like gauge theory [30, 31]. In the framework of either EQQM or a low-energy Higgs-field model it leads to the CSR constraints on some parameters of composite Higgs particles which are obtained in sec. 3.

In sec. 4 we show that CSR at high energies is indeed realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point and it is compatible with existence of relatively light scalar, h^0, and pseudoscalar, A, Higgs bosons. If the CSR constraints are fulfilled one can make further speculations about that such models can have their origin from a underlying theory with vector gauge interaction and an electroweak compositeness scale of order $1 \div 10$ TeV. The models may serve to predict the bulk of properties of Higgs particles and their couplings to fermion and gauge bosons. Some advantages and disadvantages of EQQM are discussed in sec. 5.

Let us remind the effective quark lagrangean of a EQQM which incorporates all higher-dimensional operators necessary for the description of Two-Higgs Doublet Standard Model in the low-energy limit. The two-flavor, third generation quark models with quasilocal interaction are considered in which the t- and b-quarks are involved in the DCSB.

We restrict ourselves by examination of the Two–Higgs Quark Models of type I [24, 32] with Quasilocal interaction (2HQQM) with the following lagrangean:

$$\begin{aligned}\mathcal{L}_I = &\bar{q}_L \not{D} q_L + \bar{t}_R \not{D} t_R + \bar{b}_R \not{D} b_R + \\ &+ \frac{1}{N_c \Lambda^2} \sum_{k,l=1}^{2} a_{kl} \left(g_{t,k} J_{t,k}^T + g_{b,k} \widetilde{J}_{b,k}^T \right) i\tau_2 \left(g_{b,l} J_{b,l} - g_{t,l} \widetilde{J}_{t,l} \right) .\end{aligned} \tag{1.1}$$

Here we have introduced the notations for doublets of fermion currents:

$$J_{t,k} \equiv \bar{t}_R f_{t,k}(\hat{s})\, q_L\,, \qquad J_{b,k} \equiv \bar{b}_R f_{b,k}(\hat{s})\, q_L\,, \qquad q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, \tag{1.2}$$

and the tilde in $\widetilde{J}_{t,k}$ and $\widetilde{J}_{b,k}$ marks charge conjugated quark currents, rotated with Pauli matrix τ_2,

$$\widetilde{J}_{t,k} = i\tau_2 J_{t,k}^\star\,, \qquad \widetilde{J}_{b,k} = i\tau_2 J_{b,k}^\star\,. \tag{1.3}$$

The subscripts t, b indicate right components of t and b quarks in the currents, the index k enumerates the formfactors:[1]

$$\begin{aligned} &f_{t,1}(\hat{s}) = 2 - 3\hat{s}\,; \qquad f_{t,2}(\hat{s}) = -\sqrt{3}\hat{s}\,; \qquad \hat{s} \equiv -\frac{\partial^2}{\Lambda^2}\,; \\ &f_{b,1}(\hat{s}) = 2 - 3\hat{s}\,, \qquad f_{b,2}(\hat{s}) = -\sqrt{3}\hat{s}\,; \end{aligned} \tag{1.4}$$

which are orthonormal on the interval $0 \le s = \langle \hat{s} \rangle \le 1$. In these notations coupling constants of the four-fermion interaction are represented by 2×2 matrix a_{kl} and contributed also from the Yukawa constants $g_{t,l}, g_{b,k}$.

2. Effective potential and mass spectrum of composite Higgs bosons

For sufficiently strong four-fermion coupling constants the lagrangean (1.1) reveals the phenomenon of dynamical chiral symmetry breaking due to generation of dynamical quark mass. This phenomenon is easier to investigate when the effective potential is known for the attractive scalar channel where Higgs bosons arise as composite states. Indeed a dynamical mass is induced by four-fermion forces due to a non-zero v.e.v. of quark scalar density which serves also as an interpolating operator

[1] We notice that EQQM of type II has the lagrangean [24] similar to (1.1) but with transposition of formfactors $f_{b,1} \leftrightarrow f_{b,2}$

JHEP06(1999)003

for composite scalar bosons. Thereby just a non-trivial minimum of the effective potential in the scalar channel gives rise to a dynamical mass and the perturbative fluctuations around this minimum characterize the mass spectrum of boson states.

In order to describe the dynamics of composite Higgs bosons in details the lagrangean density (1.1) of the Model I must be rearranged by means of incorporation of auxiliary bosonic variables and by integrating out fermionic degrees of freedom [20]. Namely, we define two scalar $\mathrm{SU}(2)_L$-isodoublets:

$$\Phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}, \tag{2.1}$$

and their charge conjugates:

$$\widetilde{\Phi}_1 = i\tau_2 \Phi_1^* = \begin{pmatrix} \phi_{12}^\star \\ -\phi_{11}^\star \end{pmatrix}, \qquad \widetilde{\Phi}_2 = \begin{pmatrix} \phi_{22}^\star \\ -\phi_{21}^\star \end{pmatrix}. \tag{2.2}$$

Then the lagrangean (1.1) can be rewritten in the following way:

$$\mathcal{L}_I = L_{kin} + N_c \Lambda^2 \sum_{k,l=1}^{2} \Phi_k^\dagger (a^{-1})_{kl} \Phi_l + i \sum_{k=1}^{2} \left[g_{t,k} \widetilde{\Phi}_k^\dagger J_{t,k} + g_{b,k} \Phi_k^\dagger J_{b,k} \right] + \text{h.c.} \tag{2.3}$$

The integrating out of fermionic degrees of freedom produces the effective action for Higgs bosons of which we shall keep only the kinetic term and the effective potential consisting of two- and four-particles operators. The omitted terms are supposedly small, being proportional to inverse powers of a large scale factor Λ and/or a large $\ln(\Lambda/v)$; $v \simeq 246$ GeV. The Yukawa constants are chosen of the form:

$$g_{t,k} = 1\,; \qquad g_{b,k} = g\,; \tag{2.4}$$

for $k = 1, 2$. The first choice can be done because the fields Φ_1 and Φ_2 can always be rescaled by an arbitrary factor which is absorbed by redefinition of polycritical coupling constants a_{kl}. Other constants $g_{b,k}$ are taken equal for a simplicity. Their value g induces the quark mass ratio $m_b/m_t \sim g$.

A systematic approximation can be developed in the vicinity of (poly)critical point,

$$8\pi^2 a_{kl}^{-1} \sim \delta_{kl} + \frac{\Delta_{kl}}{\Lambda^2}, \qquad |\Delta_{kl}| \ll \Lambda^2, \tag{2.5}$$

which signifies the cancellation of quadratic divergences [33].

JHEP06(1999)003

Then the effective potential of Higgs fields in 2HQQM of type I reads:

$$\begin{aligned} V_{eff} = \frac{N_c}{8\pi^2}\Bigg(&- \sum_{k,l=1}^{2} (\Phi_k^\dagger \Phi_l)\Delta_{kl} - 8g^4(\Phi_1^\dagger\Phi_1)^2 \ln g^2 + \\ &+ (1+g^4)\Bigg[8(\Phi_1^\dagger\Phi_1)^2\left(\ln\frac{\Lambda^2}{4(\Phi_1^\dagger\Phi_1)} + \frac{1}{2}\right) - \\ &\qquad - \frac{159}{8}(\Phi_1^\dagger\Phi_1)^2 + \frac{9}{8}(\Phi_2^\dagger\Phi_2)^2 + \frac{3}{4}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \\ &\qquad + \frac{3}{4}(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{3}{8}(\Phi_1^\dagger\Phi_2)^2 + \frac{3}{8}(\Phi_2^\dagger\Phi_1)^2 - \\ &\qquad - \frac{5\sqrt{3}}{4}(\Phi_1^\dagger\Phi_1)\left((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\right) + \\ &\qquad + \frac{\sqrt{3}}{4}(\Phi_2^\dagger\Phi_2)\left((\Phi_1^\dagger\Phi_2) + (\Phi_2^\dagger\Phi_1)\right)\Bigg]\Bigg) + O\left(\frac{\ln\Lambda}{\Lambda^2}\right), \end{aligned} \tag{2.6}$$

where the bilinear "mass" term is in general non-diagonal and represented by the real, symmetric 2×2 matrix Δ_{kl}. This is the two-Higgs potential in the large N_c approach and a more realistic potential should involve the true Renormalization Group flow of Two-Higgs Doublet SM[2] with certain initial conditions at high energies.

In general there exist the regimes where at the minimum of the effective potential the ratio of v.e.v. of neutral Higgs fields is complex [33] and v.e.v. of charged components are also non-zero. Due to $U(2)$ symmetry of the effective potential (2.6) one can always choose the parameterization of Higgs fields in the vicinity of a minimum with only one of v.e.v. being complex:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \theta \\ \phi_2 + i\rho \end{pmatrix}. \tag{2.7}$$

The appearance of a complex v.e.v. leads to CP violation [2, 35, 36, 37] and a non-zero v.e.v. of charged fields breaks electric stability of the vacuum and supplies the photon with a mass. The latter phase is not realized in the Universe at zero temperatures as it follows from the severe bounds on both electric charge asymmetry and on the photon mass (see, e.g. [38]).

The condition of minimum of the potential (2.6) brings the mass-gap equations for them which solution may cause the DCSB if it is an absolute minimum. In the explicit form they are:

$$\begin{aligned} \frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k,\rho,\theta)}{\delta\phi_1} &= -\Delta_{11}\phi_1 - \Delta_{12}\phi_2 + 8\phi_1^3\ln\frac{\Lambda^2}{2\phi_1^2} + 8\phi_1^3 g^4\ln\frac{\Lambda^2}{2g^2\phi_1^2} + \\ &+ \frac{1+g^4}{8}\Big[-159\phi_1^3 - 15\sqrt{3}\phi_1^2\phi_2 + 9\phi_1\phi_2^2 + \sqrt{3}\phi_2^3 + \\ &\qquad + \sqrt{3}(\theta^2+\rho^2)(\sqrt{3}\phi_1 + \phi_2)\Big] = 0, \end{aligned} \tag{2.8}$$

[2]see the updated one-loop RG equations in [34] and references therein.

$$\frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k,\rho,\theta)}{\delta\phi_2} = -\Delta_{12}\phi_1 - \Delta_{22}\phi_2 + \frac{1+g^4}{8}\Big[-5\sqrt{3}\phi_1^3 + 9\phi_1^2\phi_2 + $$
$$+3\sqrt{3}\phi_1\phi_2^2 + 9\phi_2^3 + \sqrt{3}(\theta^2+\rho^2)(\phi_1 + 3\sqrt{3}\phi_2)\Big] = 0$$
$$\frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k,\rho,\theta)}{\delta\theta} = \theta\left[-\Delta_{22} + \frac{1+g^4}{8}\Big(9(\theta^2+\rho^2) + (\sqrt{3}\phi_1+\phi_2)^2 + 8\phi_2^2\Big)\right] = 0$$
$$\frac{8\pi^2}{N_c}\frac{\delta V_{eff}(\phi_k,\rho,\theta)}{\delta\rho} = \rho\left[-\Delta_{22} + \frac{1+g^4}{8}\Big(9(\theta^2+\rho^2) + (\sqrt{3}\phi_1+\phi_2)^2 + 8\phi_2^2\Big)\right] = 0\,.$$

We assume the electric charge stability of the vacuum, i.e. that only neutral components of both Higgs doublets may have nonzero v.e.v. [3, 6, 39]. Evidently, the relevant solutions with $\theta = \rho = 0$ are unique if

$$\Delta_{22} \le (1+g^4)\left[\phi_2^2 + \frac{1}{8}(\sqrt{3}\phi_1+\phi_2)^2\right]. \tag{2.9}$$

This bound follows also from the absence of tachyons among composite bosons which will be derived later on. On the other hand, due to $SO(2)$ symmetry of effective potential (2.6) under rotations of θ and ρ, typically, the CP violating solutions for 2HQQM of type I break also the electroneutrality of the vacuum unless a special fine-tuning is performed. It gives us one more argument against the realization of CP-violating phase in such models.

One can choose ϕ_1, ϕ_2 as independent scales of 2HQQM. Then the mass-gap equations (2.9) allow to find the parameters Δ_{11}, Δ_{12} describing the deviation from critical coupling constants as functions of $\phi_1, \phi_2, \Delta_{22}$.

The true minimum is derived from the positivity of the second variation of the effective action around a solution of the mass-gap equation,

$$\Phi_k = \begin{pmatrix} \pi_k^+ \\ \frac{1}{\sqrt{2}}(\phi_k + \sigma_k + i\pi_k) \end{pmatrix}, \qquad k = 1,2\,. \tag{2.10}$$

This variation reads:

$$\frac{16\pi^2}{N_c}S^{(2)}_{eff} \equiv \frac{1}{2}\Big(\sigma, (\hat{A}^\sigma p^2 + \hat{B}^\sigma)\sigma\Big) +$$
$$+\Big(\pi^-, (\hat{A}^{\pi^\pm} p^2 + \hat{B}^{\pi^\pm})\pi^+\Big) + \frac{1}{2}\Big(\pi, (\hat{A}^\pi p^2 + \hat{B}^\pi)\pi\Big), \tag{2.11}$$

where two symmetric matrices - for the kinetic term $\hat{A}^i = (A^i_{kl})\,, \quad i = (\sigma, \pi^+, \pi)$ and for the constant, momentum independent part, $\hat{B}^i = (B^i_{kl})$ - have been introduced in the CP conserving phase.

The mass spectrum of related bosonic states is determined by the solutions of the secular equations :

$$\det(\hat{A}^i p^2 + \hat{B}^i) = 0\,, \tag{2.12}$$

at $-m^2 = p^2 < 0$ in both scalar and pseudoscalar channels.

The kinetic matrix $\hat{A}$ as being multiplied by p^2 is derived in the soft-momentum expansion in powers of p^2:

$$\mathcal{L}_{\rm kin} = \frac{N_c}{16\pi^2}\sum_{k,l=1}^{2}\left(I^{(1)}_{kl}\partial_\mu\pi^-_k\,\partial_\mu\pi^+_l + \frac{1}{2}I^{(2)}_{kl}\left(\partial_\mu\sigma_k\partial_\mu\sigma_l + \partial_\mu\pi_k\partial_\mu\pi_l\right)\right), \tag{2.13}$$

where $I^{(1)}_{kl}$ contributes to the kinetic term for charged components of Higgs doublets and $I^{(2)}_{kl}$ defines the latter one for neutral components:

$$\begin{aligned} A^{\sigma}_{kl} &\simeq A^{\pi^\pm}_{kl} \simeq A^{\pi}_{kl} \simeq I^{(1)}_{kl} \simeq I^{(2)}_{kl} \\ &\simeq f_{t,k}(0)f_{t,l}(0)\left(\ln\frac{\Lambda^2}{m_t^2}-1\right) + g^2 f_{b,k}(0)f_{b,l}(0)\left(\ln\frac{\Lambda^2}{m_b^2}-1\right) + \\ &\quad + \int_0^1 \Big(f_{t,k}(s)f_{t,l}(s) + g^2 f_{b,k}(s)f_{b,l}(s) - \\ &\qquad - f_{t,k}(0)f_{t,l}(0) - g^2 f_{b,k}(0)f_{b,l}(0)\Big)\frac{ds}{s} + O\left(\frac{\ln\frac{\Lambda^2}{m_t^2}}{\Lambda^2}\right). \end{aligned} \tag{2.14}$$

The related integrals for $I^{(1),(2)}_{kl}$ have been calculated at large values of Λ, in the large-log limit $\ln(\Lambda^2/m_t^2) \gg 1$ and for the CP conserving phase. Further on we consider $g \ll 1$ as $m_b \ll m_t$. Accordingly, in the matrix elements containing large $\ln(\Lambda^2/m_t^2)$ we neglect isospin breaking effects. After substitution of (1.4) kinetic matrices $\hat{A}$ take the form:

$$A^{\sigma}_{kl} \simeq A^{\pi}_{kl} \simeq A^{\pi^\pm}_{kl} \simeq \begin{pmatrix} \left(4\ln\frac{\Lambda^2}{m_t^2} - \frac{23}{2}\right) & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{3}{2} \end{pmatrix}. \tag{2.15}$$

Let us now obtain the matrix of second variations $\hat{B}^i$ of the effective potential for the Model I :

$$\begin{aligned} B^{\sigma}_{kl} &= \frac{16\pi^2}{N_c}\frac{\partial^2}{\partial\sigma_k\partial\sigma_l}V_{eff}\,, \qquad B^{\pi}_{kl} = \frac{16\pi^2}{N_c}\frac{\partial^2}{\partial\pi_k\partial\pi_l}V_{eff}\,, \\ B^{\pi^\pm}_{kl} &= \frac{8\pi^2}{N_c}\frac{\partial^2}{\partial\pi^+_k\partial\pi^-_l}V_{eff}\,, \qquad \text{for } \Phi_k = \langle\Phi_k\rangle. \end{aligned} \tag{2.16}$$

In the above approximation: $B^{\pi^\pm}_{kl} \simeq B^{\pi}_{kl}$ and therefore the spectra of charged and neutral pseudoscalars coincide. Other elements of matrices $\hat{B}$ are:

$$\begin{aligned} B^{\sigma}_{11} &= -2\Delta_{11} + 48\phi_1^2\ln\left(\frac{\Lambda^2}{m_t^2}\right) - \frac{605}{4}\phi_1^2 - \frac{15\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\phi_2^2\,, \\ B^{\sigma}_{12} &= -2\Delta_{12} - \frac{15\sqrt{3}}{4}\phi_1^2 + \frac{9}{2}\phi_1\phi_2 + \frac{3\sqrt{3}}{4}\phi_2^2\,, \\ B^{\sigma}_{22} &= -2\Delta_{22} + \frac{9}{4}\phi_1^2 + \frac{3\sqrt{3}}{2}\phi_1\phi_2 + \frac{27}{4}\phi_2^2\,, \end{aligned}$$

$$
\begin{aligned}
B^{\pi}_{11} &= -2\Delta_{11} + 16\phi_1^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right) - \frac{159}{4}\phi_1^2 - \frac{5\sqrt{3}}{2}\phi_1\phi_2 + \frac{3}{4}\phi_2^2\,, \\
B^{\pi}_{12} &= -2\Delta_{12} - \frac{5\sqrt{3}}{4}\phi_1^2 + \frac{3}{2}\phi_1\phi_2 + \frac{\sqrt{3}}{4}\phi_2^2\,, \\
B^{\pi}_{22} &= -2\Delta_{22} + \frac{3}{4}\phi_1^2 + \frac{\sqrt{3}}{2}\phi_1\phi_2 + \frac{9}{4}\phi_2^2\,.
\end{aligned} \tag{2.17}
$$

For large $\ln(\Lambda^2/m_t^2) \gg 1$ both the solutions of (2.9) and the mass spectrum look differently in two regimes: $\Delta_{11} \sim m_t^2 \ln(\Lambda^2/m_t^2) \gg \Delta_{12} \sim \Delta_{22} \sim m_t^2$ or $\Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \sim m_t^2 \ln(\Lambda^2/m_t^2) \gg m_t^2$.

In the first case the mass spectrum of physical scalar bosons H^0, h^0 reads:

$$
\begin{aligned}
m^2_{H^0} &= 8\phi_1^2 = 4m_t^2 \simeq \frac{\Delta_{11}}{\ln(\Lambda^2/m_t^2)}\,; \\
m^2_{h^0} &= \frac{1}{2}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{3}{2}\delta m^2 - \frac{4}{3}\Delta_{22}\,,
\end{aligned} \tag{2.18}
$$

and for physical pseudoscalar bosons $\pi, A, H^{\pm}$:

$$
m^2_{\pi} = 0\,; \qquad m^2_A = m^2_{H^{\pm}} = \frac{1}{6}\left(8\phi_2^2 + (\sqrt{3}\phi_1 + \phi_2)^2\right) - \frac{4}{3}\Delta_{22} \equiv \frac{1}{2}\delta m^2 - \frac{4}{3}\Delta_{22}\,, \tag{2.19}
$$

where we have introduced the notation for the mass splitting between excited states:

$$
m^2_{h^0} - m^2_A = \delta m^2\,. \tag{2.20}
$$

One can see that, in general, $m^2_{h^0} - m^2_A = \delta m^2 \neq m^2_{H^0} - m^2_{\pi}$ and the splitting between scalar and pseudoscalar states is not equidistant. But if the dynamical mass is a true constant, i.e. $\sqrt{3}\phi_1 + \phi_2 = 0$, then one recovers the remarkable relation:

$$
m^2_{h^0} - m^2_A = m^2_{H^0} - m^2_{\pi} = 4m_t^2\,. \tag{2.21}
$$

As $m_t^2 = 2\phi_1^2$ the parameter ϕ_1 is fixed from the phenomenology and never takes zero values. Then for a given Δ_{22} the lowest masses of excited bosons arise at $\phi_1 = -3\sqrt{3}\phi_2$ and amount to:

$$
\begin{aligned}
\delta m^2_{min} &= \frac{1}{9} m^2_{H^0}\,; \qquad m^2_{h^0} = \frac{1}{6} m^2_{H^0} - \frac{4}{3}\Delta_{22} \geq \frac{1}{9} m^2_{H^0}\,; \\
m^2_A &= \frac{1}{18} m^2_{H^0} - \frac{4}{3}\Delta_{22} \geq 0\,.
\end{aligned} \tag{2.22}
$$

It follows from these inequalities that the second pseudoscalar Higgs boson can in principle be very light even massless whereas the second scalar Higgs particle can be lighter than the first one (of Nambu-Jona-Lasinio type) but still enough heavy.

For instance, if we treat these relations at the tree level then: $m_{H^0} = 2m_t \simeq$ 350 GeV but the lightest mass for the second Higgs boson $m^2_{h^0} = \frac{1}{9} m^2_{H^0} + m^2_A \geq (130\text{GeV})^2$ for $m_A \geq 50$ GeV (see [40]).

In the case when $m_t^2 \ln(\Lambda^2/m_t^2) \sim \Delta_{11} \sim \Delta_{12} \sim \Delta_{22} \gg m_t^2$ the mass spectra can be easily derived from eqs. (2.18), (2.19) retaining the last term, $m_{h^0}^2 \simeq m_A^2 \simeq -\frac{4}{3}\Delta_{22}$ and their splitting is again described by (2.20). Thus for such parameters Δ_{ik} the second multiplet of Higgs boson is indeed much heavier that the first one $m_{H^0}^2 \ll m_{h^0}^2 \simeq m_A^2 \sim m_t^2 \ln(\Lambda^2/m_t^2) \sim (1\mathrm{TeV})^2$. Hence this scenario is practically equivalent to a One-Higgs SM for accessible energies. Therefore we will not consider it further on.

Thus we have outlined the mass spectrum in EQQM generated due to CSB and established the range of four-fermion coupling constants which promotes the formation of relatively light Higgs bosons. On the other hand CSB occurs at low energies and one should expect the Chiral Symmetry Restoration for correlators of quark currents when in-, out-coming 4-momenta are sufficiently high. The pattern of CSR, of course, depends on the structure of underlying theory which causes a four-fermion interaction below the compositeness scale. In the next Section we show what a pattern of CSR is characteristic for underlying gauge theories with vector coupling to quarks. As well we derive the relevant CSR constraints on parameters of scalar and pseudoscalar resonances when they are composite quark-antiquark states bound due to a gauge interaction.

3. Chiral symmetry restoration in vector-like gauge models

Let us consider the two-point correlators of scalar and pseudoscalar quark densities which are sensitive to CSB. Namely, in the large N_c approach and for the vanishing current quark mass, they consist of a sum of meson poles, with different positions and residues for scalar and pseudoscalar channels, and of a perturbative contribution, $C(p^2)$, the same in both channels:

$$\begin{aligned}
\Pi_\sigma(p^2) &= -\int d^4x\, \exp(ipx)\, \langle T\,(\bar{q}q(x)\; \bar{q}q(0))\rangle \\
&= \sum_n \frac{Z_n^\sigma}{p^2 + m_{\sigma,n}^2} + C(p^2), \\
\Pi_\pi^{ab}(p^2) &= \int d^4x\, \exp(ipx)\, \langle T\left(\bar{q}\gamma_5\tau^a q(x)\; \bar{q}\gamma_5\tau^b q(0)\right)\rangle \\
&= \delta^{ab}\left(\sum_n \frac{Z_n^\pi}{p^2 + m_{\pi,n}^2} + C(p^2)\right).
\end{aligned} \tag{3.1}$$

In large N_c QCD-like models [41, 42] the leading contributions into these two-point correlators are given solely by sums over the infinite number of meson poles whereas $C(p^2) = C_0^\pi + C_1^\pi p^2$ is polynomial with C_0 and C_1 being contact terms required for the regularization of infinite sums.

On the other hand, the high-energy asymptotics of the correlators (3.1) for non-Abelian gauge theories are given by perturbation theory and OPE taking into account the asymptotic freedom of gauge interaction. As well the nonperturbative generation of gauge field and quark condensates [30] is assumed to determine subleading power-like corrections to perturbative asymptotics which, in particular, parameterize CSB.

Let us denote as $g_s^{(j)}$ the coupling constants of gauge interactions generating four-fermion forces and eventually, composite bosons. Then the conventional coupling constants are $\bar{\alpha}_s^{(j)} \equiv g_s^{(j)}/4\pi$. Here the index $j = 1, 2, \ldots$ enumerates different (mostly heavy) vector field species (for instance, in the Kaluza-Klein scenario when more spatial compactified dimensions are present [26]). We assume that the gauge interactions are non-abelian and vector fields are represented in the algebra of SU(N_c), $G_\mu^{(j)} \equiv g_s^{(j)} \lambda^a G_\mu^{(j)a}$ For every type of gauge fields one defines $\mathrm{tr}(\lambda^a \lambda^b) = 2\delta^{ab}$. For a simplification, we restrict ourselves by only one type of gauge field providing four-fermion forces at low energies and, from now on, omit the index j.

In the chiral limit ($m_q = 0$) CSB is unambiguously described by the difference between the scalar and pseudoscalar correlators which have the following behavior [30, 31, 43] at large p^2 (for one-flavor quarks) motivated by OPE (in Euclidean notations):

$$\left(\Pi_\pi(p^2) - \Pi_\sigma(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right);$$
$$\Delta_{SP} = -\frac{\pi\bar{\alpha}_s}{p^4} \langle \bar{q}\sigma_{\mu\nu}\lambda^k q \bar{q}\sigma_{\mu\nu}\lambda^k q\rangle \simeq 12\pi\bar{\alpha}_s \left(\langle \bar{q} q\rangle\right)^2, \qquad (3.2)$$

where the vacuum dominance hypothesis [30] (in the large-N_c limit) has been exploited to estimate four-quark condensate contribution in terms of bilinear quark condensates $< \bar{q}q >$. It is also assumed that $\bar{\alpha}_s(\Lambda_{CSB}) \ll 1$.

This rapidly decreasing asymptotics is a consequence of the chiral invariance of the lagrangean of a vector gauge theory with massless fermions. We stress that all perturbative contributions and nonperturbative terms induced solely by gluons or by resummation of perturbative series are completely canceled between two correlators as they do not break the chiral symmetry.

On the other hand, due to CSR, the saturation in the difference of scalar and pseudoscalar correlators may be successfully delivered by a finite number of low-lying resonances [18, 28].

Thus CSR at high energies may be thought of as a possible constraint on the EQQM to be a manifestation of compositeness in the Higgs model [33, 44]. We shall demand that, at the compositeness scale Λ_{CSB}, the relation (3.2) were approximately fulfilled.

When being inspired by the planar limit of the underlying gauge interaction, eq. (3.1), one can make the two-resonance ansatz for scalar and pseudoscalar correlators at intermediate energies supported by the Two-channel EQQM:

$$\Pi_\sigma(p) = \frac{Z_\sigma}{p^2 + m_\sigma^2} + \frac{Z_{\sigma'}}{p^2 + m_{\sigma'}^2} + C_0^\sigma;$$
$$\Pi_\pi(p) = \frac{Z_\pi}{p^2} + \frac{Z_{\pi'}}{p^2 + m_{\pi'}^2} + C_0^\pi. \qquad (3.3)$$

We remark that for this type of models the constants can be taken $C_1^\sigma = C_1^\pi = 0$. From the requirement of asymptotic CSR (3.2) it follows that:

$$C_0^\sigma = C_0^\pi \equiv C \left(= \frac{\langle \bar{q} q \rangle}{m_{dyn}} < 0 \right); \tag{3.4}$$

$$Z_\sigma + Z_{\sigma'} = Z_\pi + Z_{\pi'}\,; \qquad Z_\sigma m_\sigma^2 + Z_{\sigma'} m_{\sigma'}^2 = Z_{\pi'} m_{\pi'}^2 + \Delta_{SP}\,. \tag{3.5}$$

The first two relations can be fulfilled in the conventional NJL model which corresponds to the one-resonance ansatz, $Z_{\sigma',\pi'} = 0$, whereas the last one can be provided only in a two-resonance model, for the Δ_{SP} of (3.2) (see below).

Let us apply the above CSR sum rules to the 2HQQM (1.1). For this goal we have to obtain the appropriate two-point correlators for scalar and pseudoscalar composite Higgs fields.

4. CSR sum rules in 2HQQM

Let us introduce in 2HQQM the external sources which couple to the scalar and pseudoscalar quark densities. With their help one can derive required quark correlators. As these densities are doublets in our model, eq. (1.2), the relevant sources $X_1 = (\chi_{1j}), X_2 = (\chi_{2j})$ are also taken as doublets. The sources are complex: $\chi_{kj} = S_{kj} + iP_{kj}$, generating both scalar and pseudoscalar densities. The structure of the corresponding operators in the quark lagrangean is designed similar to the Yukawa operators in (2.3):

$$\mathcal{L}_I(S,P) = i \sum_{k=1}^{2} \left[g_{t,k} \widetilde{X}_k^\dagger J_{t,k} + g_{b,k} X_k^\dagger J_{b,k} \right] + \text{h.c.} \tag{4.1}$$

We remind that in 2HQQM under consideration: $g_{t,k} = 1 \gg g_{b,k} = g$ and take for a simplicity $g = 0$. Then the effect of external sources can be separated by shifting the Higgs fields in the quark part of the lagrangean (2.3):

$$\phi_{kj} \longrightarrow \bar{\phi}_{kj} + \chi_{kj}\,. \tag{4.2}$$

The dynamical boson fields arise as fluctuations around the solutions of the mass-gap equation (2.9):

$$\bar{\phi}_{k1} = \phi_{k1} - \chi_{k1}; \quad \bar{\phi}_{k2} = \phi_{k2} - \chi_{k2} + \langle \bar{\phi}_{k2} \rangle\,. \tag{4.3}$$

In terms of doublets X_k, Φ_k the supplementary lagrangean takes the form:

$$\begin{aligned}\Delta \mathcal{L}_I = N_c \Lambda^2 \sum_{k,l=1}^{2} \Big[& X_k^\dagger (a^{-1})_{kl} X_l - X_k^\dagger (a^{-1})_{kl} \Phi_l - \\ & - \Phi_k^\dagger (a^{-1})_{kl} X_l - X_k^\dagger (a^{-1})_{kl} \langle \Phi_l \rangle - \langle \Phi_k^\dagger \rangle (a^{-1})_{kl} X_l \Big]\,. \end{aligned} \tag{4.4}$$

The terms linear in external sources are irrelevant for correlators and will be neglected further on. As we study the CP conserving phase with non-trivial v.e.v. only for

JHEP06(1999)003

scalar fields the physical parameters of neutral and charged pseudoscalar bosons are the same. Therefore we can consider neutral components only. Then with the notations:

$$\phi_{k2} = \frac{1}{\sqrt{2}}(\sigma_k + i\pi_k)\,; \qquad \chi_{k2} = \frac{1}{\sqrt{2}}(S_k + i\,P_k)\,, \tag{4.5}$$

one obtains the quadratic part of the lagrangean $\mathcal{L}_I$ consisted of (2.11) and (4.4). As it is Gaussian one can unravel the dependence on external fields with the help of classical equations of motion:

$$\begin{aligned}
\sigma_k^{\mathrm{cl}} &= 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl} a^{-1}_{lm} S_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl} S_l \\
\pi_k^{\mathrm{cl}} &= 16\pi^2\Lambda^2 \sum_{l,m=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl} a^{-1}_{lm} P_m \simeq 2\Lambda^2 \sum_{l=1}^{2} \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl} P_l\,,
\end{aligned} \tag{4.6}$$

which have been reduced in the vicinity of polycritical point, $8\pi^2 a^{-1}_{kl} \simeq \delta_{kl}$.

The resulting effective action for generating of two-point correlators is given by:

$$\begin{aligned}
S^{(2)} &\simeq \frac{N_c\Lambda^2}{16\pi^2} \sum_{k,l=1}^{2} \left(S_k \Gamma^{(\sigma)}_{kl} S_l + P_k \Gamma^{(\pi)}_{kl} P_l\right) \\
\Gamma^{(\sigma)}_{kl} &= \delta_{kl} - 2\Lambda^2 \left(A^{\sigma\sigma}p^2 + B^{\sigma\sigma}\right)^{-1}_{kl}\,; \qquad \Gamma^{(\pi)}_{kl} = \delta_{kl} - 2\Lambda^2 \left(A^{\pi\pi}p^2 + B^{\pi\pi}\right)^{-1}_{kl}\,.
\end{aligned} \tag{4.7}$$

When taking the approximate expressions (2.15), (2.17) for matrices $\hat{A}, \hat{B}$ one derives the inverse propagators. In particular, the strictly local quark density can be presented as a superposition of two currents:

$$\bar{t}t = \frac{1}{2}\left(\bar{t}f_1 t - \sqrt{3}\bar{t}f_2 t\right). \tag{4.8}$$

Respectively, the related correlator (3.1) in the scalar channel has the following form:

$$\begin{aligned}
\Pi_\sigma(p^2) &= -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma^{(\sigma)}_{11} + 3\Gamma^{(\sigma)}_{22} - 2\sqrt{3}\Gamma^{(\sigma)}_{12}\right] = C^\sigma + \frac{Z_{H^0}}{p^2 + m^2_{H^0}} + \frac{Z_{h^0}}{p^2 + m^2_{h^0}}\,; \\
C^\sigma &= -\frac{N_c\Lambda^2}{4\pi^2} \simeq \frac{\langle\bar{t}t\rangle}{m_t}\,; \\
Z_{H^0} &= \frac{N_c\Lambda^4}{48\pi^2\left(\ln\frac{\Lambda^2}{m_t^2} - 3\right)\left(m^2_{H^0} - m^2_{h^0}\right)} \times \\
&\quad \times \left[12\left(\ln\frac{\Lambda^2}{m_t^2} - 3\right) m^2_{H^0} + 6\Delta_{11} + 4\sqrt{3}\Delta_{12} + 2\Delta_{22} - 144\phi_1^2 \ln\frac{\Lambda^2}{m_t^2} + \right. \\
&\quad \left. + 474\phi_1^2 + 12\sqrt{3}\phi_1\phi_2 - 18\phi_2^2\right] = O\left(\frac{1}{\ln\frac{\Lambda^2}{m_t^2}}\right); \\
Z_{h^0} &= -Z_{H^0} + \frac{N_c\Lambda^4}{4\pi^2}\,;
\end{aligned} \tag{4.9}$$

and that one in the pseudoscalar channel reads:

$$\Pi_\pi(p^2) = -\frac{N_c\Lambda^2}{16\pi^2}\left[\Gamma^{(\pi)}_{11} + 3\Gamma^{(\pi)}_{22} - 2\sqrt{3}\Gamma^{(\pi)}_{12}\right] = C^\pi + \frac{Z^\pi}{p^2} + \frac{Z_A}{p^2+m_A^2}\,;$$

$$C^\pi = C^\sigma = -\frac{N_c\Lambda^2}{4\pi^2} \simeq \frac{\langle\bar{t}t\rangle}{m_t};$$

$$Z^\pi = -\frac{N_c\Lambda^4}{48\pi^2\left(\ln\frac{\Lambda^2}{m_t^2}-3\right)\,m_A^2}\left[2\Delta_{22} + \Delta_{12}\left(4\sqrt{3} - 6\frac{\phi_2}{\phi_1}\right) + \frac{27}{4}\phi_1^2 - \frac{29\sqrt{3}}{4}\phi_1\phi_2 + \right.$$

$$\left. + \frac{3}{4}\phi_2^2 + \frac{3\sqrt{3}}{4}\frac{\phi_2^3}{\phi_1}\right] = O\left(\frac{1}{\ln\frac{\Lambda^2}{m_t^2}}\right)\,;$$

$$Z_A = -Z_\pi + \frac{N_c\Lambda^4}{4\pi^2}\,. \tag{4.10}$$

We stress that the residues in poles are of different order of magnitude:

$$Z_{H^0} \sim Z^\pi \sim \frac{1}{\ln\frac{\Lambda^2}{m_t^2}} \ll Z_{h^0} \sim Z_A\,, \tag{4.11}$$

which can be seen from the first equation in (4.9) after using the mass-gap eqs. (2.9) and taking the value (2.18) of the scalar boson mass, $m_{H^0}^2 \simeq 8\phi_1^2$. Indeed then the logarithms cancel each other in the combination, $12m_{H^0}^2 \ln\frac{\Lambda^2}{m_t^2}m_{H^0}^2 + 6\Delta_{11} - 144\phi_1^2\ln\frac{\Lambda^2}{m_t^2}$.

Now we are able to impose and check the CSR constraints (3.5). First of all one can see that the chiral symmetry is not broken in leading asymptotics, $C^\pi = C^\sigma$. Next we check the subleading asymptotics which represents the generalized σ-model relation:

$$Z_{H^0} + Z_{h^0} = Z_\pi + Z_A = \frac{N_c\Lambda^4}{4\pi^2}\,, \tag{4.12}$$

which is fulfilled, in fact, at a high precision including subleading $1/\ln\Lambda^2$ terms.

The last constraint in (3.5) can be satisfied for an appropriate 4-fermion condensate contribution Δ_{SP}. Thus:

$$Z_{H^0}m_{H^0}^2 + Z_{h^0}m_{h^0}^2 = Z_A m_A^2 + \Delta_{SP}\,. \tag{4.13}$$

Taking into account (4.11) and the sum rule (4.12) one concludes that:

$$Z_{h^0} \simeq Z_A \simeq \frac{N_c\Lambda^4}{4\pi^2}; \qquad \frac{N_c\Lambda^4}{4\pi^2}\left(m_{h^0}^2 - m_A^2\right) \simeq \Delta_{SP}\,. \tag{4.14}$$

In order to estimate a maximal scale of compositeness $\Lambda \equiv \Lambda_C$ due to vector-like forces we assume that presumably the typical scale of gauge interactions is of order $\mu \sim m_t \ll \Lambda_C$ and one can apply effectively the one-loop approximation to calculate the strong interaction constant $\bar{\alpha}_s$ (see, [40]). As it follows from (2.22), the minimal value of mass splitting corresponds to $m_{H^0}^2/9$. Respectively the value of Δ_{SP} is described by the relation (3.2) and in the present 2HQQM the condensates are given in (4.9). Let us combine these estimates and relations to derive the upper bound on the compositeness scale Λ_C:

$$\frac{N_c \Lambda_C^4 m_t^2}{9\pi^2} = \min(\Delta_{SP}) = 12\pi\bar{\alpha}_s \frac{N_c^2 \Lambda_C^4 m_t^2}{16\pi^4}\,; \tag{4.15}$$

hence:

$$\bar{\alpha}_s N_c \simeq \frac{12\pi}{11\left(1 - \frac{2N_f}{N_c}\right)\ln\frac{\Lambda_C^2}{m_t^2}} > \frac{4\pi}{27} \quad \text{or} \quad \Lambda_C \leq 10^4\text{GeV}\,. \tag{4.16}$$

Thereby with the help of CSR sum rules we have shown that there is a certain window to have relatively light composite Higgs bosons in the effective 2HQQM created by a more fundamental vector-like theory with a pretty low compositenes scale of order of 10 TeV.

5. Conclusions

1. We have found that in the framework of 2HQQM of type I two composite Higgs doublets can be created dynamically as a consequence of DCSB in two channels. Lighter Higgs bosons appear as radial excited states in the language of the Potential Quark Model.

2. We have proved that CSR at high energies is realized in the 2HQQM of type I in the Nambu-Jona-Lasinio phase near tricritical point. Thereby these models in the NJL phase can be regarded as effective models originating from a underlying vector-gauge theory with an electroweak compositeness scale of order $1 \div 10$ TeV (it may be also a fundamental, quantum gravity or string scale [26, 29] when additional spatial dimensions of inverse size > 10 Mev exist).

3. One can show that in other phases explored in two-channel models in [20, 33], such as the anomalous or special phase, the last CSR constraint cannot be fulfilled for *any* choice of parameters. It means that such phases cannot be realized in effective models motivated by underlying vector-like gauge theory. But the question remains open about what kind of underlying theory could induce such phases at lower energies.

4. It turns out that the soft-momentum limit of the correlators (3.3) is connected to the structural constants of the effective chiral lagrangean [45, 46] (in our case of the EW lagrangean in the large Higgs mass limit [47]):

$$\frac{Z_{H^0}}{m_{H^0}^2} + \frac{Z_{h^0}}{m_{h^0}^2} + C = 8B_0^2(2L_8 + H_2)\,;$$
$$\frac{Z_A}{m_A^2} + C = 8B_0^2(-2L_8 + H_2)\,. \tag{5.1}$$

One can think about how to apply these relations together with the empirical estimations on the corresponding structural coupling constant for the determination of the size of the top-quark condensate $\langle \bar{q}q \rangle = -B_0 F_0^2$ (with F_0 being an electroweak scale).

5. For a comparison we also check the consistency of the last sum rule (4.13) in the one-channel top-condensate SM. One deals then with only one scalar, H^0, and a triplet of Goldstone bosons, π^a. Respectively, in the sum rules (3.5), (4.13) one should retain only Z^π, Z_{H^0}. Hence they read:

$$Z^\pi = Z_{H^0} = \frac{N_c \Lambda^4}{2\pi^2 \ln \frac{\Lambda^2}{m_t^2}} \, ;$$
$$Z_{H^0} m_{H^0}^2 = 2 \frac{N_c \Lambda_C^4 m_t^2}{\pi^2 \ln \frac{\Lambda_C^2}{m_t^2}} \neq \Delta_{SP} = \frac{9}{11} \frac{N_c \Lambda_C^4 m_t^2}{\pi^2 \ln \frac{\Lambda_C^2}{m_t^2}} \, , \tag{5.2}$$

where eqs. (4.7) with $A \simeq A_{11}, B \simeq B_{11}$ from (2.15), (2.17) are employed, the Nambu relation $m_{H^0}^2 = 4m_t^2$ is used and the correlators are derived according to eqs. (4.9), (4.10). As well the definition (3.2) for Δ_{SP} and the expression (4.16) for $\bar{\alpha}_s$ for a SU(N_c) gauge field is inserted. Evidently there is a discrepancy between the left and right sides of the last sum rule which is remarkably independent on scales, colors and flavors. Thus it happens to be improbable to embed the one-channel Top-mode SM into a simple underlying gauge theory of QCD type. On the other hand it is of interest to find an appropriate set of scalar and vector gauge fields which could fit well the relation between $Z_{H^0} m_{H^0}^2$ and Δ_{SP}.

6. In this paper we have not considered the CP and electroneutrality violating scenario for 2HQQM in details although it may occur as an intermediate stage in the formation of baryon asymmetry of the Universe [38, 48].

7. For low compositeness scales ~ 10 TeV the v.e.v.'s of these supposedly lightest Higgs fields do not saturate the EW scale $v = 246$ GeV, for phenomenological values of the t-quark mass. Thereby these fields provide only a part of the full Higgs mechanism which requires more (heavy) scalar fields [16] to induce the observable masses of W, Z bosons. Another possibility to provide the masses of W, Z bosons when having a relatively light t quark may be to extend the quark sector of 2HQQM with a seventh quark, just modifying the so called top condensation seesaw mechanism [22].

Acknowledgments

We express our gratitude to DFG for financial support which makes it possible to continue our research on properties of composite bosons in effective quark models. We are also grateful to D. Espriu for useful discussion and invaluable remarks and to an anonymous referee for relevant criticism.

References

[1] A.A. Anselm, N.G. Uraltsev and V.A. Khoze, *Sov. Phys. Usp.* **28** (1985) 113;
A.A. Anselm, *Surveys High Energ.Phys.* **7** (1994) 107.

[2] T.D. Lee, *Phys. Rev.* **D 8** (1973) 1226.

[3] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley, 1990.

[4] S.L. Glashow and S. Weinberg, *Phys. Rev.* **D 15** (1977) 1958.

[5] D. Atwood, L. Reina and A. Soni, *Phys. Rev.* **D 55** (1997) 3156 [hep-ph/9609279].

[6] G. Kreyerhoff and R. Rodenberg, *Phys. Lett.* **B 226** (1989) 323;
J. Freund, G. Kreyerhoff and R. Rodenberg, *Phys. Lett.* **B 280** (1992) 267.

[7] M. Sher, *Phys. Rep.* **179** (1989) 273;
S. Nie and M. Sher, *Phys. Lett.* **B 449** (1999) 89 [hep-ph/9811234].

[8] J. Erler and P. Langacker, *Status of Standard Model*, hep-ph/9809352.

[9] G. Degrassi et al, *Phys. Lett.* **B 418** (1998) 209 [hep-ph/9708311];
G. D'Agostini and G. Degrassi, *Constraints on the Higgs boson mass from direct searches and precision measurements*, hep-ph/9902226.

[10] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.

[11] V.A. Miransky, M. Tanabashi and K. Yamawaki, *Mod. Phys. Lett.* **A 4** (1989) 1043; *Phys. Lett.* **B 221** (1989) 177.

[12] W.J. Marciano, *Phys. Rev. Lett.* **62** (1989) 2793; *Phys. Rev.* **D 41** (1990) 219.

[13] W.A. Bardeen, C.T. Hill and M. Lindner, *Phys. Rev.* **D 41** (1990) 1647.

[14] M.A. Luty, *Phys. Rev.* **D 41** (1990) 2893.

[15] M. Suzuki, *Phys. Rev.* **D 41** (1990) 3457.

[16] M. Lindner and D. Lüst, *Phys. Lett.* **B 272** (1991) 91;
M. Lindner, *Int. J. Mod. Phys.* **A 8** (1993) 2167.

[17] S. Weinberg, *Phys. Rev. Lett.* **18** (1967) 507.

[18] A.A. Andrianov and V.A. Andrianov, *Zapiski Nauch. Sem. POMI (Proc. Steklov Math. Inst., St. Petersburg, Russia)*, **245**, No.14 (1996) 5 [hep-ph/9705364].

[19] M. Krawczyk, *Testing 2HDM at muon colliders*, hep-ph/9803484; *Constraints on the Higgs sector from processes involving photons*, hep-ph/9812536 and refs. therein.

[20] A.A. Andrianov and V.A. Andrianov, *Int. J. Mod. Phys.* **A 8** (1993) 1981; *Theor. Math. Phys.* **93** (1992) 1126; *Theor. Math. Phys.* **94** (1993) 3; *Effective fermion models in symmetry–breaking phase and quantum chromodynamics*, hep-ph/9309297, Proc. School-Sem. *Hadrons and nuclei from QCD*, Tsuruga/Vladivostok/Sapporo 1993, WSPC, 1994, 341; *Zap. Nauch. Sem. POMI (Proc. Steklov Math. Inst.,St. Petersburg, Russia)*, **209**, No.12 (1994) 3; *Nucl. Phys.* **39BC** *(Proc. Suppl.)* (1995) 257.

[21] K.G. Wilson and J.B. Kogut, *Phys. Rep.* **C12** (1974) 75.

[22] B.A. Dobrescu and C.T. Hill, *Phys. Rev. Lett.* **81** (1998) 2634 [hep-ph/9712319].

[23] R.S. Chivukula, B.A. Dobrescu, H. Georgi and C.T. Hill, *Phys. Rev.* **D 59** (1999) 075003 [hep-ph/9809470] and references therein.

[24] A.A. Andrianov, V.A. Andrianov, V.L. Yudichev and R. Rodenberg, *Two Higgs doublet model from quasilocal quark interactions*, hep-ph/9709475; *Int. J. Mod. Phys.* **A 14** (1999) 323.

[25] T.L. Barklow et al., *Strong coupling electroweak symmetry breaking*, 1996 DPF/DPB Summer Study, Snowmass, Colorado, hep-ph/9704217.

[26] B.A. Dobrescu, *Electroweak symmetry breaking as a consequence of compact dimensions*, hep-ph/9812349.

[27] C.T. Hill, *Phys. Lett.* **B 266** (1991) 419.

[28] A.A. Andrianov, D. Espriu and R. Tarrach, *Nucl. Phys.* **B 533** (1998) 429 [hep-ph/9803232].

[29] T. Banks, A. Nelson and M. Dine, *J. High Energy Phys.* **06** (1999) 014 [hep-th/9903019].

[30] M.A. Shifman, A.I. Vainstein and V.I. Zakharov, *Nucl. Phys.* **B 147** (1979) 385, *Nucl. Phys.* **B 147** (1979) 448.

[31] L.J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rep.* **127** (1985) 1 and references therein.

[32] G. Cvetic, *Rev. Mod. Phys.* **71** (1999) 513 [hep-ph/9702381] and references therein.

[33] A.A. Andrianov, V.A. Andrianov and V.L. Yudichev, *Dynamical P-parity breaking in effective quark model*, hep-ph/9512256, proc. X Int. Worksh. on HEP and QFT (Zvenigorod, Russia, Sept. 1995), MSU Publ., 1996, 211; *Composite two-Higgs model with dynamical CP-violation*, hep-ph/9610376, proc. XI Int. Worksh. on HEP and QFT (St.Petersburg, Sept. 1996), MSU Publ., 1997, 160; *Theor. Math. Phys.* **108** (1996) 1069.

[34] G. Cvetic, C.S. Kim and S.S. Hwang, *Phys. Rev.* **D 58** (1998) 116003 [hep-ph/9806282].

[35] G.C. Branco, *Phys. Rev.* **D 22** (1980) 2901;
G.C. Branco, J.-M. Gerard and W. Grimus, *Phys. Lett.* **B 136** (1984) 383.

[36] A. Skjold and P. Osland, *Nucl. Phys.* **B 453** (1995) 3 [hep-ph/9502283];
C.A. Boe et al. *Higgs production: CP studies in e-e- collisions*, hep-ph/9811505.

[37] W. Bernreuter, *CP violation*, hep-ph/9808453 and refs. therein;
W. Bernreuter, A. Brandenburg and M. Flesch, *Phys. Rev.* **D 56** (1997) 90 [hep-ph/9701347]; *Effects oh Higgs sector CP violation in top-quark pair production at the LHC*, hep-ph/9812387.

[38] A.B. Lahanas, V.C. Spanos and V. Zarikas, *Electric charge breaking in two stage phase transitions of models with two Higgs doublets*, hep-ph/9812535.

[39] R. Rodenberg, *Int. J. Mod. Phys.* **A 11** (1996) 4779 and references therein.

[40] C.Caso *et al.* (PDG), *Eur. Phys. J.* **C 3** (1998) 1.

[41] G. t'Hooft, *Nucl. Phys.* **B 72** (1974) 461.

[42] E. Witten, *Nucl. Phys.* **B 160** (1979) 57.

[43] M. Jamin and M. Münz, *Z. Physik* **C 66** (1995) 633 [hep-ph/9409335].

[44] N.V. Krasnikov and R. Rodenberg, *Nuovo Cim.* **A105** (1992) 1601.

[45] J. Gasser and H. Leutwyler. *Nucl. Phys.* **B 250** (1985) 465.

[46] J. Bijnens, E. de Rafael and H. Zheng, *Z. Physik* **C 62** (1994) 437 [hep-ph/9306323];
J. Bijnens, *Phys. Rep.* **265** (1996) 369 [hep-ph/9502335].

[47] A. Dobado, D. Espriu and M.J. Herrero, *Phys. Lett.* **B 255** (1991) 405;
P. Ciafaloni and D. Espriu, *Phys. Rev.* **D 56** (1997) 1752 [hep-ph/9612383].

[48] M. Laine and K. Rummukainen, *Nucl. Phys.* **B 545** (1999) 141 [hep-ph/9811369].